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On the complexification of Minkowski spacetime

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ABSTRACT

It is well known that any two arbitrary observers S and S' moving relative to each other with a speed v< c in isotropic space see a 4-dimensional real spacetime. We demonstrate that the two observers should naturally see the spacetime as a complexified 4-dimensional manifold described by the Kähler manifold commonly studied in string theory. Such a complex spacetime has, on large scales, been demonstrated to be a natural consequence of special relativity when quantum effects are included in relativistic mechanics and are thus of much significance in quantum gravity, quantum super string theory, particle physics and cosmology

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1. Introduction

The dimensional structure of spacetime is as fundamental to cosmology as are dynamics of matter and radiation. In the standard model, spacetime is 3+1 real and becomes Minkowskian if time is imaginary. An infinite dimensional (Cantorian) spacetime E^{∞} with topological dimension n_t can be realized in a complex spacetime manifold class belonging to the Kähler manifold in string theory [1]. A formal definition of complex time T and its complex conjugate $T*$ was first proposed by El Naschier [2] where the imaginary time was interpreted as "past" time, the complex conjugate as "future" time and their intersection given by the modulus

$$t = \sqrt{TT*}. \quad (1.1)$$

represents the time "now" where $T*$ denotes the complex conjugate. Further investigation of complex time by Mejias [2,3,5] proposed the two-dimensional time

$$T = t' + i\left(v/c\right)t. \quad (1.2)$$

which relates the times t and t' respectively in two inertial frames S and S' in relative motion with speed $v < c$ such that putting (1.2) and its complex conjugate into (1.1) yields the well-known time dilation in special relativity

$$t = t'\left(1 - v^2/c^2\right)^{1/2}. \quad (1.3)$$

In this paper we demonstrate that spacetime in one frame of reference necessarily becomes complexified in a second inertial frame in a manner that preserves the well known Lorentz transformations in special relativity.

2. Complex spacetime

We consider a rocket fired with a speed v from a point A in a given direction such that after time t the rocket is at point R_θ as shown in figure 1. We suppose that a light signal is released simultaneously with the rocket and travels a distance ct_{AD} to reach a detector at some point D.

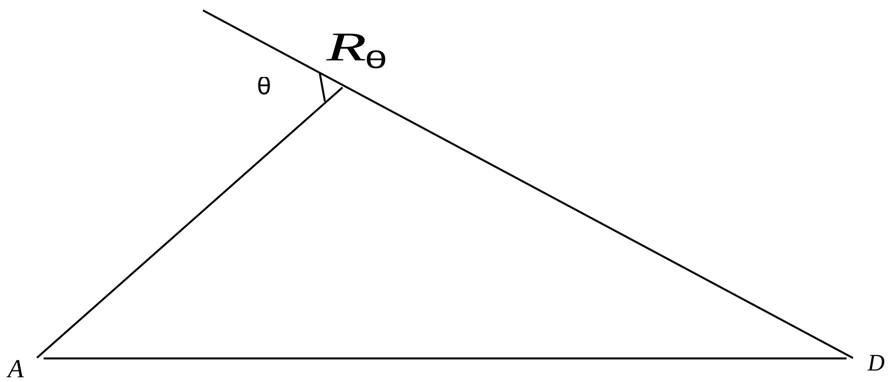

Figure 1. Locus of D and R_θ about A after time t in isotropic space.

Considering vectors measured from point A (in the S frame) as AD and AR_θ, and those from point R_θ (in the S' frame) as $R_\theta D$ and $R_\theta A$ then in general the three points A, R_θ and D form a triangle such that by cosine rule we have

$$AD^2 = AR_\theta{}^2 + DR_\theta{}^2 + 2\tilde{A}R_\theta \tilde{R}_\theta D\cos\theta. \quad (2.1)$$

or equivalently,

$$(c^2 - v^2)t_A{}^2 = R_\theta D^2 + 2c\tilde{R}_\theta D\cos\theta t_{\tilde{A}D}. \quad (2.2)$$

Considering that the distance

$$R_\theta D = ct_{R_\theta D}. \quad (2.3)$$

Where $t_{R_\theta D}$ is the time light takes to cover the distance $R_\theta D$ then by the rocket clock, (2.2) becomes

$$(c^2 - v^2)\gamma_\theta{}^2 = c^2 + 2c^2\cos\theta\gamma_\theta. \quad (2.4)$$

Where γ_θ is defined by the relation

$$t_{\tilde{A}D} = \gamma_\theta t_{\tilde{R}_\theta D}. \quad (2.5)$$

Setting the values of θ equal to 0, $\pi/2$ and π in (2.4) yields

$\gamma_\pi \gamma_0 = \gamma_{\pi/2} \equiv \gamma^2$. (2.6)

and,

$\gamma_0 \gamma_\theta^{\ 2} = \gamma_0 \gamma^2 + (\gamma_0^{\ 2} - \gamma^2)\gamma_\theta \cos\theta$. (2.7)

Where γ is the Lorentz factor of special relativity. We hence have the solution to (2.7) as

$\gamma_0 = \dfrac{(\gamma_0^{\ 2} - \gamma^2)\cos\theta \pm \sqrt{(\gamma_0^{\ 2} - \gamma^2)^2 + 4\gamma_0^{\ 2}\gamma^2}}{2\gamma_0}$. (2.8)

and consequently

$\gamma_0 = \dfrac{(\gamma_0^{\ 2} - \gamma^2) \pm (\gamma_0^{\ 2} + \gamma^2)}{2\gamma_0}$. (2.9)

whose nontrivial solution is given by

$\gamma_0^{\ 2} = -\gamma^2$. (2.10)

Substituting (2.10) into (2.8) yields

$t_{\tilde{A}D} = Z_c t_{\tilde{R}_\theta D}$. (2.11)

Where Z_c is a complex number given by

$Z_c = i\gamma e^{\pm i\theta} \equiv i\gamma(\cos\theta \pm i\sin\theta)$. (2.12)

This then implies that t is complex time.

Similarly, it can be shown that the distance $R_\theta D$ takes the complex form,

$|R_\theta D| = \dfrac{e^{\pm i\theta}}{i\gamma} ct$. (2.13)

and as such the 4-dimensional spacetime is fully complex.

3. The boost parameter

From Fif.1, it is clear that both the observers S and S' agree that $|\tilde{A}R_\theta|^2 = |\tilde{R}_\theta A|^2$ but would not agree on the modulus $|R_\theta D|^2$ since from point A it follows that

$R_\theta D = AD - AR_\theta$. (3.1)

yet we have

$|\tilde{R}_\theta D| \geq |\tilde{A}D| - |\tilde{A}R_\theta|$. (3.2)

Now, we can define a scaling factor γ such that (3.2) transforms to

$$|\tilde{R}_\theta D| = \gamma(|\tilde{A}D| - |\tilde{A}R_\theta|). \quad (3.3)$$

The scaling factor γ can be interpreted as the Lorentz factor with the Lorentz transformations given by the usual expressions

$$|\tilde{R}_\theta D| = \gamma(|\tilde{A}D - vt_A|). \quad (3.4)$$
$$|\tilde{A}D| = \gamma(|\tilde{R}_\theta D| + vt_{R_\theta D}). \quad (3.5)$$
$$t_{\tilde{R}_\theta D} = \gamma(-v/c^2\,|\tilde{A}D| + t_{\tilde{A}D}). \quad (3.6)$$
$$t_{\tilde{A}D} = \gamma(v/c^2\,|R_\theta D| + t_{R_\theta D}). \quad (3.7)$$

Combining (2.5), (3.6) and (3.7) it becomes apparent that

$$v/c = [i\,e^{\pm i\theta} - 1]. \quad (3.8)$$

which is the boost parameter $\beta = v/c$ in complex form.

4. Discussion and conclusion

In this paper, we have shown that complex spacetime arises naturally from a generalized transformation and derived the complex boost parameter in isotropic space. Our results are in agreement with earlier studies by [4, 5] and display a straight-forward approach to understanding the widely studied complexification of space and time, important in the unification of the four fundamental forces of nature [4, 6].

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